Book of Abstracts

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List of Abstracts

Existence of equilibria for 3D neo-Hookean materials

Duvan Henao

Facultad de Matemáticas, Pontificia Universidad Católica de Chile

Whenever the stored energy density of a hyperelastic material has slow growth at infinity (below $C|F|^p$ with p less than the space dimension), it may undergo cavitation (the nucleation and sudden growth of internal voids) under large hydrostatic tension [Ball, 1982; James & Spector, 1992]. This constitutes a failure of quasiconvexity and, hence, a challenge for the existence theory in elastostatics [Ball & Murat, 1984]. The obstacle has been overcome under certain coercivity hypotheses [Mller & Spector, 1995; Sivaloganathan & Spector, 2000] which, however, fail to be satisfied by the paradigmatic example in elasticity: that of 3D neo-Hookean materials. A joint work with Marco Barchiesi, Carlos Mora-Corral, and Rémy Rodiac will be presented, where this borderline case was solved for hollow axisymmetric domains. Partial results leading to a solution when the axis of rotation is contained (where the dipoles found by [Conti & De Lellis, 2003] must be proved to be non energy-minimizing) will also be discussed.

Non-standard behaviour of bent periodic plates for subcritical thickness-period scalings

Kirill Cherednichenko

Department of Mathematical Sciences, University of Bath, UK

I will discuss nonlinearly elastic plates of thickness $h \to 0$ with an ε-periodic structure such that $\varepsilon^{-2}h \to 0$. I will show that in this asymptotic regime the plate exhibits non-standard behaviour in the asymptotic two-dimensional reduction from three-dimensional elasticity: in general, its effective stored-energy density is "discontinuously anisotropic" in all directions. The proof relies on a new result concerning an additional isometric constraint that deformation fields must satisfy on the microscale.

On the 3D Ginzburg-Landau model of superconductivity

Carlos Román

Facultad de Matemáticas, Pontificia Universidad Católica de Chile

The Ginzburg-Landau model is a phenomenological description of superconductivity. A crucial feature is the occurrence of vortices (similar to those in fluid mechanics, but quantized), which appear above a certain value of the strength of the applied magnetic field called the first critical field. In this talk I will present a sharp estimate of this value and describe the behavior of global minimizers for the 3D Ginzburg-Landau functional below and near it.

This is partially joint work with Etienne Sandier and Sylvia Serfaty.

The complex dynamics of Faraday pilot waves: a hydrodynamic quantum analogue

Paul Milewski

Department of Mathematical Sciences, University of Bath, UK

Faraday pilot waves are a newly discovered hydrodynamic structure that consists a bouncing droplet which creates, and is propelled by, a Faraday wave. These pilot waves can behave in extremely complex ways exhibiting a classical form of wave-particle duality, and result in dynamics mimicking quantum mechanics, including multiple quantisation, probabilistic particle distributions reminiscent of QM, diffraction and tunnelling. I will show some of this fascinating behaviour and will develop a surface wave-droplet fluid model that captures many of the features observed, and focus on rationalising the emergence of the statistics of complex states.

How Bloch waves can understand Hashim-Shtrikman microstructures

Carlos Conca

Centro de Modelamiento Matemático (CMM), Departamento de Ingeniería Matemática, Universidad de Chile

In this lecture we apply spectral methods by using the Bloch waves to study the homogenization process in the non-periodic class of generalized Hashin-Shtrikman micro-structures (Tartar in "The General Theory of Homogenization", volume 7 of Lecture Notes of the Unione Matematica Italiana, Springer, Berlin, p. 281, 2009), which incorporates both translation and dilation with a family of scales, including one subclass of laminates. We establish the classical homogenization result providing the spectral representation of the homogenized coefficients. It offers a new lead towards extending the Bloch spectral analysis in a non-periodic, non-commutative class of micro-structures.

The sharp exponent in the study of the nonlocal Hénon equation in $\mathbb{R}^N$. A Liouville theorem and an existence result

Alexander Quaas
Departamento de Matemática, Universidad Técnica Federico Santa María, Chile

We consider the nonlocal Hénon equation

$$(-\Delta)^s u = |x|^\alpha u^p, \quad \mathbb{R}^N,$$

where $(-\Delta)^s$ is the fractional Laplacian operator with $0 < s < 1$, $-2s < \alpha$, $p > 1$ and $N > 2s$. We prove a nonexistence result for positive solutions in the optimal range of the nonlinearity, that is, when

$$1 < p < p^*_{\alpha,s} := \frac{N + 2\alpha + 2s}{N - 2s}.$$

Moreover, we prove that a bubble solution, that is, a fast-decay positive radially symmetric solution, exists when $p = p^*_{\alpha,s}$.

This is joint work with B. Barrios.

Pattern formation in confined nematic systems

Apala Majumdar
Department of Mathematical Sciences, University of Bath, UK

Nematic liquid crystals are classical examples of partially ordered materials intermediate between isotropic liquids and crystalline solids. We study spatio-temporal pattern formation for nematic liquid crystals in two-dimensional regular polygons, subject to physically relevant non-trivial tangent boundary conditions, in the powerful continuum Landau-de Gennes framework. We study two asymptotic limits, relevant for “small” nano-scale domains and macroscopic domains respectively, and analytically study how the solution landscape changes with the domain size, including the admissible singularities. Notably, in the small domain limit, we always have an isolated degree $+1$ vortex at the centre of the regular polygon, which splits into fractional defects at the polygon vertices, as the domain size increases. We numerically compute bifurcation diagrams using arc continuation methods and deflation techniques, tracking stable and unstable nematic equilibria as a function of domain size. In the last part of the talk, we discuss two-dimensional ferronematic systems, as a generalization of our work on nematic equilibria in regular polygons, and the coupling between the nematic order parameter and the spontaneous magnetization induced by the suspended nanoparticles. Our most striking numerical observations concern the stabilization of interior fractional nematic point defects and magnetic domain walls, purely induced by geometric effects and the ferronematic coupling, without any external magnetic fields. These results have been obtained in collaboration with different research groups around the world and all collaborators will be acknowledged during the talk.

Energy minimizing frame fields in $\mathbb{R}^3$

Alberto Montero
Facultad de Matemáticas, Pontificia Universidad Católica de Chile

A 3-frame is an unordered orthonornal basis in $\mathbb{R}^3$. A frame field on a set $\Omega \subset \mathbb{R}^3$ is a map that assigns a 3-frame to each point in Ω. In this talk I will consider a representation of the set of 3-frames as the zero set of a finite family of polynomials in a (finite dimensional) vector space, and use this to both define and seek energy minimizing frame fields in a bounded, smooth Ω in $\mathbb{R}^3$.

This is joint work with D. Golovaty and D. Spirn.

Numerical methods for fractional diffusion

Enrique Otarola
Departamento de Matemática, Universidad Técnica Federico Santa María, Chile

We consider the spectral fractional Laplacian in bounded domains Ω and present solution techniques for the nonuniformly elliptic problem, posed on $\Omega \times (0, \infty)$, that localizes it. We establish regularity estimates for the solution of this problem; in particular, the analytic regularity with respect to the extended variable. We present a first-degree tensor product FEM and the tensorization of a first-degree FEM in Ω with a suitable hp-FEM in the extended variable.

Density interpolation methods

Carlos Pérez
Instituto de Ingeniería Matemática y Computacional (IMC), Escuela de Ingeniería, Pontificia Universidad Católica de Chile

This talk presents ongoing work on a class of effective and simple-to-implement methods for the numerical evaluation of boundary integral operators and layer potentials in two and three spatial dimensions. These methods rely on the use of Green's third identity and local Taylor-like interpolations of density functions in terms of homogeneous solutions of the underlying PDE. The proposed technique effectively regularizes the singularities present in boundary integral operators and layer potentials, and recasts the former in terms of integrands that are bounded or even more regular, depending on the order of the density interpolation. The resulting boundary integrals can be easily, accurately, and inexpensively evaluated by means of standard quadrature rules. A variety of numerical examples demonstrate the effectiveness of the technique in the context of Nyström and boundary element methods for the Laplace, Helmholtz, and Maxwell equations.

This is joint work with Catalin Turc (Department of Mathematical Sciences, NJIT) and Luiz Faria (Laboratoire POEMS, INRIA).

Symplectic Hybridizable discontinuous Galerkin methods for wave propagation problems

Manuel Sánchez

Instituto de Ingeniería Matemática y Computacional (IMC), Escuela de Ingeniería, Pontificia Universidad Católica de Chile

We devise Hybridizable discontinuous Galerkin methods, Mixed methods, and Discontinuous Galerkin methods for wave propagation problems written in Hamiltonian form. The main objective of the space discretizations is to preserve the Hamiltonian structure for a slightly modified discrete Hamiltonian defined for each method. We then discretize in time using symplectic time integrators, specifically, partitioned Runge-Kutta and diagonally implicit Runge-Kutta methods. The fundamental feature of the fully discrete schemes is that the conservation of the energy, the discrete Hamiltonian, is guaranteed. We present numerical examples that suggest optimal convergence of the errors and long-time simulations experiments that confirm the energy-conserving properties of the numerical schemes.

Self-similar, blow-up solutions of the Generalised Korteweg-de Vries equation near criticality

Chris Budd

Department of Mathematical Sciences, University of Bath, UK

In this talk we will give a detailed asymptotic analysis of the near critical self-similar blow-up solutions to the Generalized Korteweg-de Vries equation (GKdV). For a nonlinearity that has a power larger than the critical value $p = 5$, solitary waves of the GKdV can become unstable and become infinite in finite time, in other words they blow up. Numerical simulations indicate that if $p > 5$ the solitary waves travel to the right with an increasing speed, and simultaneously, form a similarity structure as they approach the blow-up time. This structure breaks down at $p = 5$. Based on these observations, we rescale the GKdV equation to give an equation that will be analysed by using asymptotic methods as p approaches 5. By doing this we resolve the complete structure of these self-similar blow-up solutions and study the singular nature of the solutions in the critical limit. In both the numerics and the asymptotics, we find that the solution has sech-like behaviour near the peak. Moreover, it becomes asymmetric with slow algebraic decay to the left of the peak and much more rapid algebraic decay to the right. The asymptotic expressions agree to high accuracy with the numerical results, performed by adaptive high-order solvers based on collocation or finite difference methods. Based on these expressions we make some conjectures about the approximately self-similar form of the solutions when $p = 5$.

Large time behavior of solutions of the porous medium equation in exterior domains

Carmen Cortázar

Facultad de Matemáticas, Pontificia Universidad Católica de Chile

Let $\mathcal{H} \subset \mathbb{R}^n$ be a non-empty bounded open set. We study the large time behavior of the porous medium equation in the complement of $\mathcal{H}$, with zero Dirichlet data on its boundary and nonnegative compactly supported integrable initial data.

This is joint work with Fernando Quirós (Universidad Autonoma de Madrid, Spain) and Noemi Wolanski (Universidad de Buenos Aires, Argentina).

Descriptions of decay for Hamiltonian $abcd$ Boussinesq system

Chulkwang Kwak

Facultad de Matemáticas, Pontificia Universidad Católica de Chile

The Boussinesq $abcd$ system was originally derived by Bona, Chen and Saut [J. Nonlinear. Sci. (2002)] as first order 2-wave approximations of the incompressible and irrotational, two dimensional water wave equations in the shallow water wave regime. Among many particular regimes, the Hamiltonian generic regime is characterized by the setting $b = d > 0$ and $a, c < 0$. It is known that the system in this regime is globally well-posed for small data in the energy space $H^1 \times H^1$ by Bona, Chen and Saut [Nonlinearity (2004)]. In this talk, we are going to discuss the decay of small solutions to $abcd$ systems in three directions. First, for a sufficiently *dispersive* $abcd$ system (characterized only in terms of the parameters a, b and c), all small solutions must decay to zero, locally strongly in the energy space, in proper subsets of the light cone $|x| \leq |t|$. Second, for every ray $x = vt$, $|v| < 1$ inside the light cone, small solutions to a sufficiently dispersive system (smallness and dispersion are characterized by v) decay to zero, in proper subsets along the ray. Last, small solutions decay to zero in exterior regions $|x| \gg |t|$ under suitable conditions of the parameters (a, b, c). All results rule out, among other things, the existence of zero or nonzero speed or super-luminical small solitary waves in each regime where decay is present.

This is joint work with Claudio Muñoz, Felipe Poblete and Juan C. Pozo.

Nontrivial solutions to Serrin's problem in annular domains

Luciano Sciaraffia

Facultad de Matemáticas, Pontificia Universidad Católica de Chile

A classical result due to Serrin establishes that if $\Omega \subset \mathbb{R}^n$ is a bounded smooth domain which admits a solution to the overdetermined boundary value problem

$$-\Delta u = 1 \quad \text{in} \quad \Omega, \qquad u = 0,\ u_\nu = \text{const} \quad \text{on} \quad \partial\Omega,$$

where ν denotes the *inner* unit normal to $\partial\Omega$, then Ω must be a ball and u is radial. If we consider the related problem for a smooth annular domain $\Omega = \Omega_1 \setminus \overline{\Omega}_1$

$$-\Delta u = 1 \quad \text{in} \quad \Omega, \quad u = 0,\ u_\nu = \text{const} \quad \text{on} \quad \partial\Omega_1, \quad u = \text{const},\ u_\nu = \text{const} \quad \text{on} \quad \partial\Omega_0,$$

and suppose that u is a positive solution that satisfies $u_\nu|_{\partial\Omega_0} \leq 0$, from results by Reichel and Sirakov it is known that Ω_0 and Ω_1 must be concentric balls and u radially symmetric. We show that the condition $u_\nu|_{\partial\Omega_0} \leq 0$ cannot be dropped by constructing non-radially symmetric smooth bounded domains $\Omega \subset \mathbb{R}^n$ of the form $\Omega_0 \setminus \overline{\Omega}_1$, bifurcating from annuli, for which there exists a positive solution to the problem. Furthemore, we show that the constructed domains are self-Cheeger.

This is joint work with Nikola Kamburov.

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p-Harmonic functions with nonlinear Neumann boundary conditions and measure data

Natham Aguirre

Facultad de Matemáticas, Pontificia Universidad Católica de Chile

In this presentation I will discuss the problem of finding p-harmonic functions on the upper half-space subject to nonlinear Neumann boundary conditions involving measures. Using the theory of renormalized solutions, we are able to obtain several existence results.

Multiplicity of solutions for some elliptic equations with a gradient term in the nonlinearity

Ignacio Guerra

Departamento de Matemática, Universidad de Santiago de Chile

We consider the problem

$$\begin{aligned} -\Delta u &= \lambda \frac{(1+|\nabla u|^q)}{(1-u)^p}, \quad 0<u<1, \quad \text{in} \quad B, \\ u &= 0 \qquad \text{on} \quad \partial B, \end{aligned}$$

where B is the unit ball in $\mathbb{R}^N$, $p>0$, $q\geq 0$ and $\lambda \geq 0$.

The problem with $q=0$ is well know. In fact, Joseph & Lundgren found that for $2<N<4\frac{p}{p+1}+4\sqrt{\frac{p}{p+1}}+2$ there are infinitely many solutions for some $\lambda=\lambda_*>0$. On the other hand, they also found that for $N\geq 4\frac{p}{p+1}+4\sqrt{\frac{p}{p+1}}+2$ there exists λ^* such that there exists a unique solution for each $0<\lambda<\lambda^*$.

In this talk, we present results of existence of solutions for this problem when $q>0$. In this case, we found a rich structure of solutions depending on p, q and the dimension N. In addition, we study numerically the behaviour of solutions for related problems with a gradient term in the nonlinearity.

On fluctuations in particle systems and their links to partial differential equations

Johannes Zimmer

Department of Mathematical Sciences, University of Bath, UK

Partial differential equations often arise as a scaling limit of particle models. We present a link that allows to determine the evolution operator of a class of parabolic equations directly from particle data. As a preliminary step, we will explain how transport coefficients in these equations can be computed from particle simulations. The argument relies on a suitable representation of the governing PDE as gradient flow of the entropy.

Specifically, we study particle systems and analyse their fluctuations. Fluctuations can often provide useful information on underlying processes, for example in form of fluctuation-dissipation relations. These fluctuations can be described by stochastic differential equations or variational formulations related to large deviations. The link between finite systems and their many-particle limit will be analysed in this formulation.

This is joint work with X. Li, P. Embacher, N. Dirr and C. Reina.

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